Fuzzy Control

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Goal

- To help researchers and engineers in the field of machine learning tackle problems in **control systems**

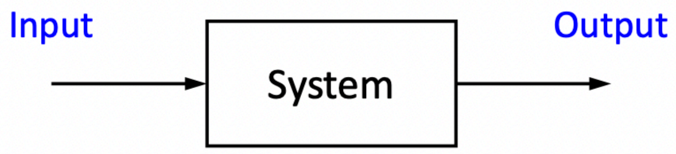
  - Control systems involve **real-time decision making**: a kind of artificial intelligence
  - Overview of **control theory** that may be helpful for proper use of machine learning

What is Control?

- To operate a **system** as desired

What is System?

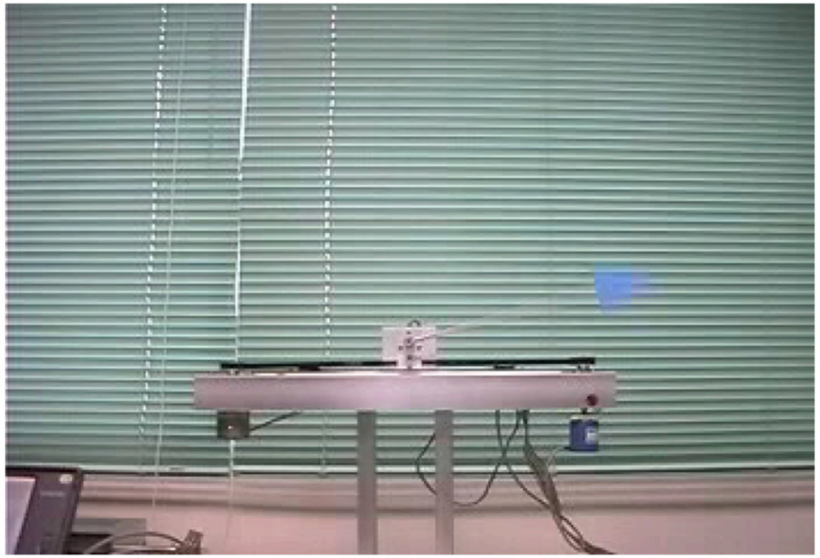
- Something changing dynamically according to inputs



Block Diagram

Input and Output:
Signals (Functions of Time)

Control Systems

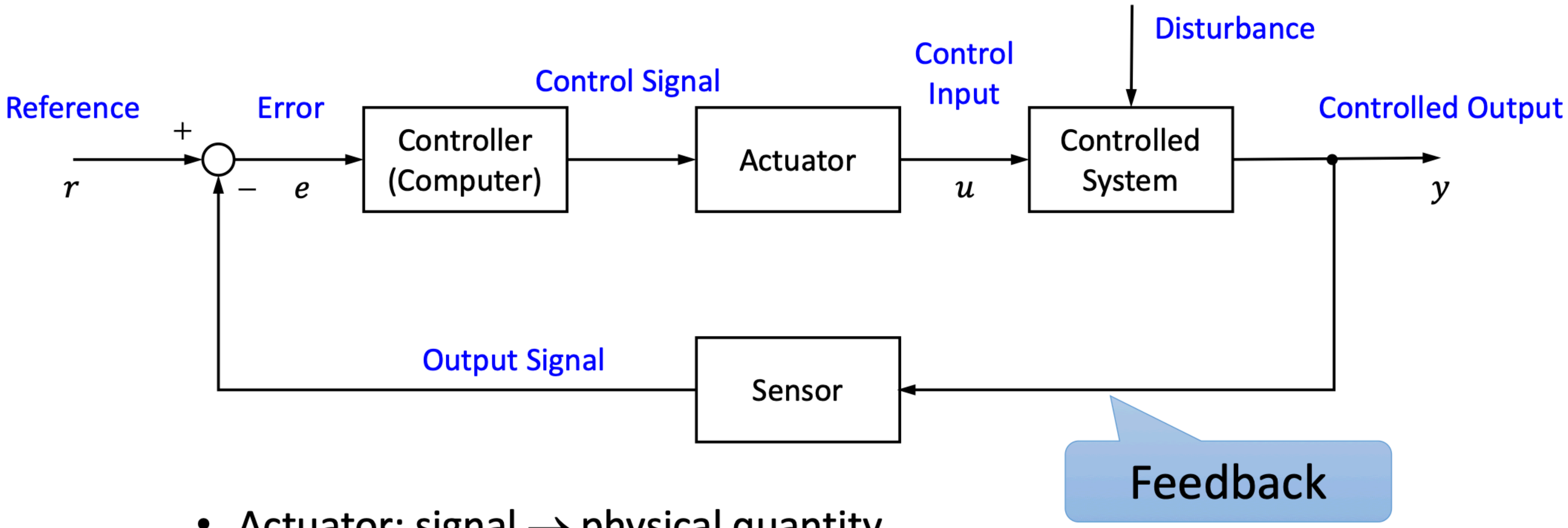


Inverted Pendulum © Toru Asai 2004

Rocket © JAXA 2014

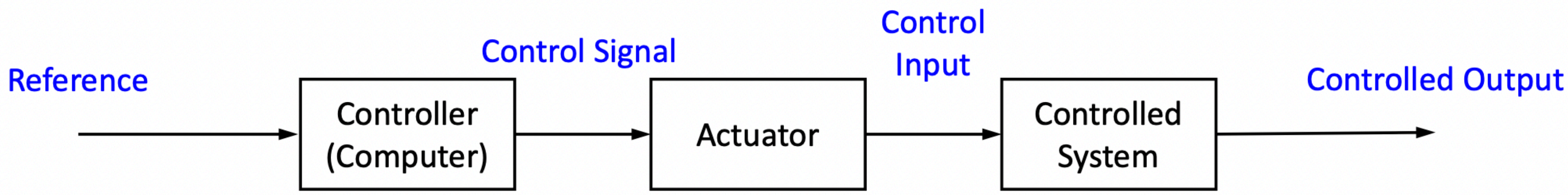
Systems kept upward by control against gravity

Feedback Control System (Closed-Loop)



- Actuator: signal → physical quantity
- Sensor: physical quantity → signal
- Actuator/sensor blocks are often omitted.

Feedforward Control System (Open-Loop)



- No sensor
- No disturbance

Control Systems are Everywhere

- Such machines as cars, ships, aircraft, and robots
  - **Inputs:** forces, torques, steering
  - **Outputs:** positions, velocities, directions
- Temperature, environment, economy, and epidemic
  - **Inputs:** heat, gas emissions, monetary policy, mask/vaccine mandate
  - **Outputs:** temperature, atmospheric constituent, money supply, spread rate

Control Engineering

- Methodology to **analyze** and **design** control systems
- Methodology based on mathematical models of control systems: **Control Theory**
- A lot of definitions, **theorems** and proofs: Stability, Controllability, Optimality, etc.

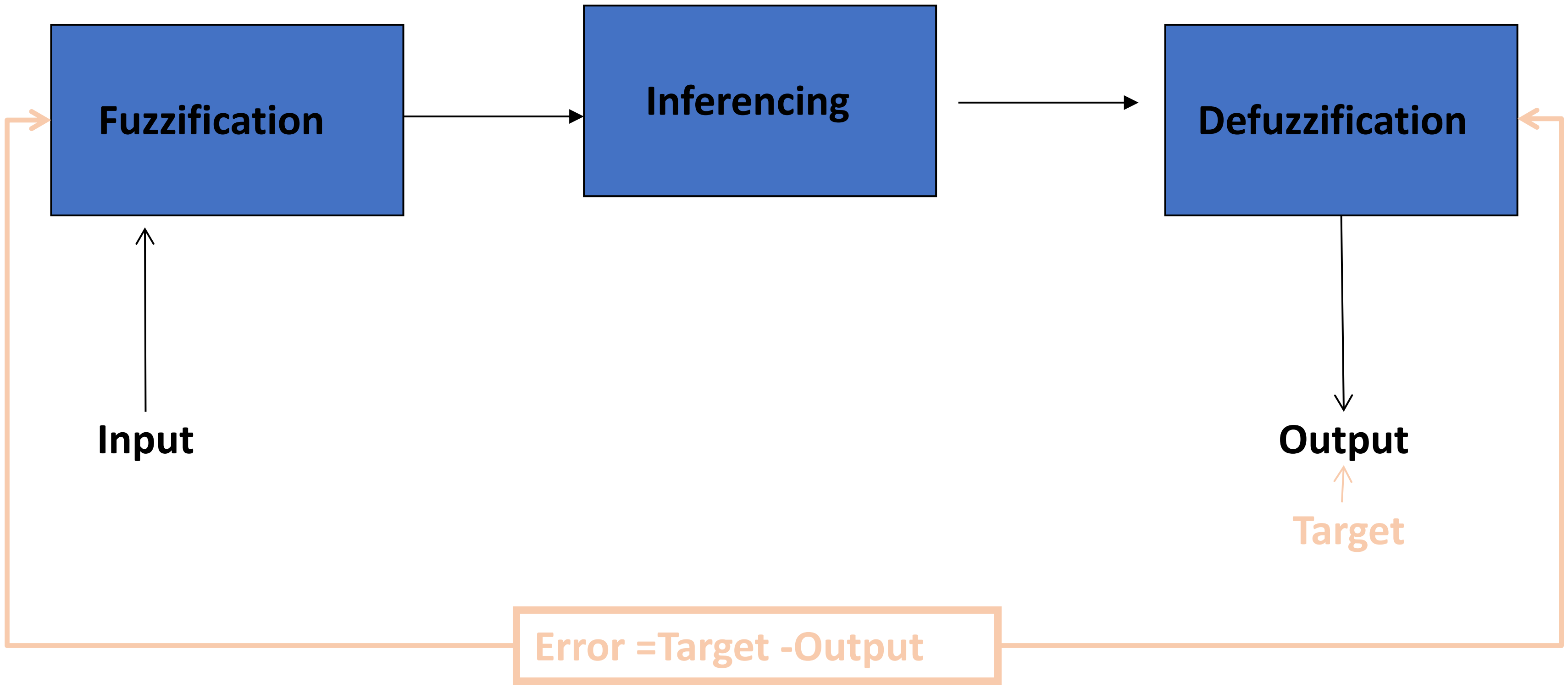
Fuzzy Perspective

- Rules Contrain MS functions
- Consequent MS function can be simplified
  - Variable space (i.e., error) → partitions
- Within each partition, output variable:
  - linear functions of the inputs
  - Nonlinear functions…
  - Not a MS function

Review Fuzzy Models

If <antecedence> then <consequence>.

Basic Configuration of a Fuzzy Logic System



Types of Rules

Mamdani Model

R1: If x is A_1 and y is B_1 then z is C_1

R2: If x is A_2 and y is B_2 then z is C_2

A_i , B_i and $C_{i,}$ are fuzzy sets defined on the universes of x, y, z respectively

Takagi-Sugeno Model

R1: If x is A_1 and y is B_1 then $z = f_1(x,y)$

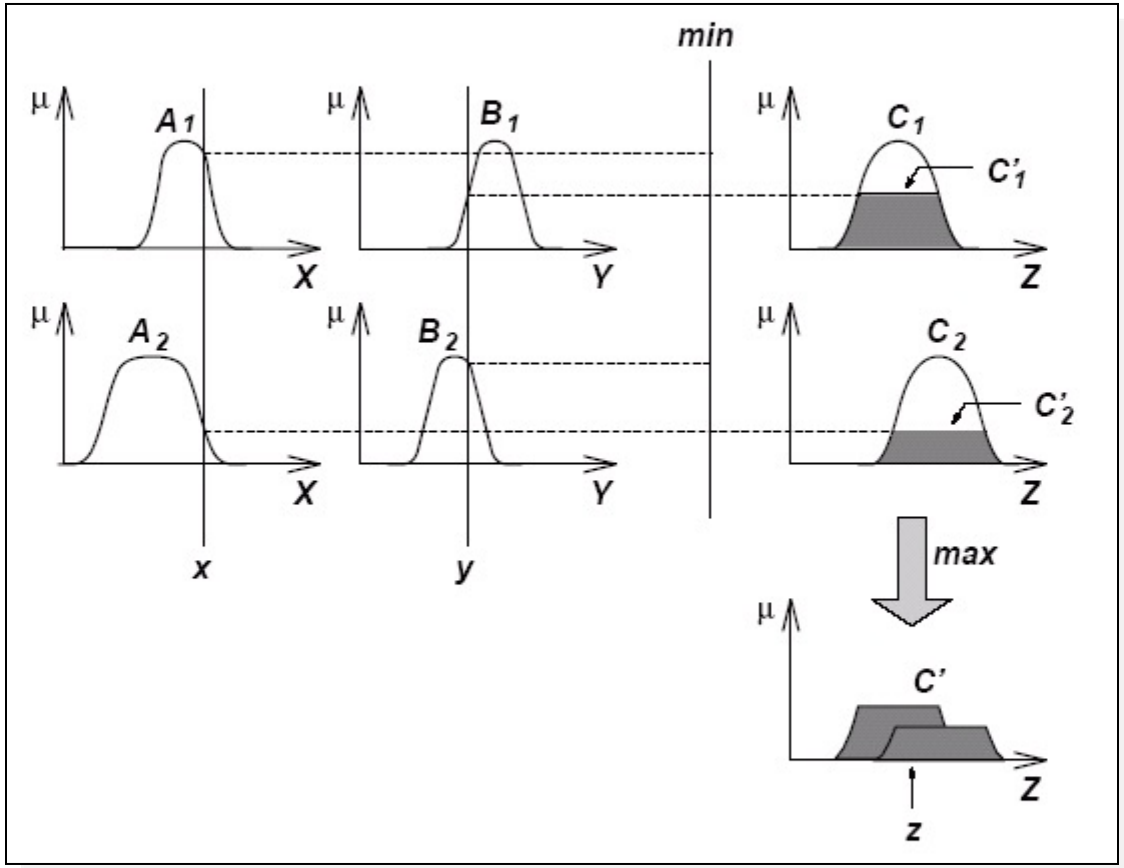
R1: If x is A_2 and y is B_2 then $z = f_2(x,y)$

For example: $f_i(x,y)=a_i x+b_i y+c_i$

Mamdani Fuzzy Models

The Reasoning Scheme

Both antecedent and consequent are fuzzy

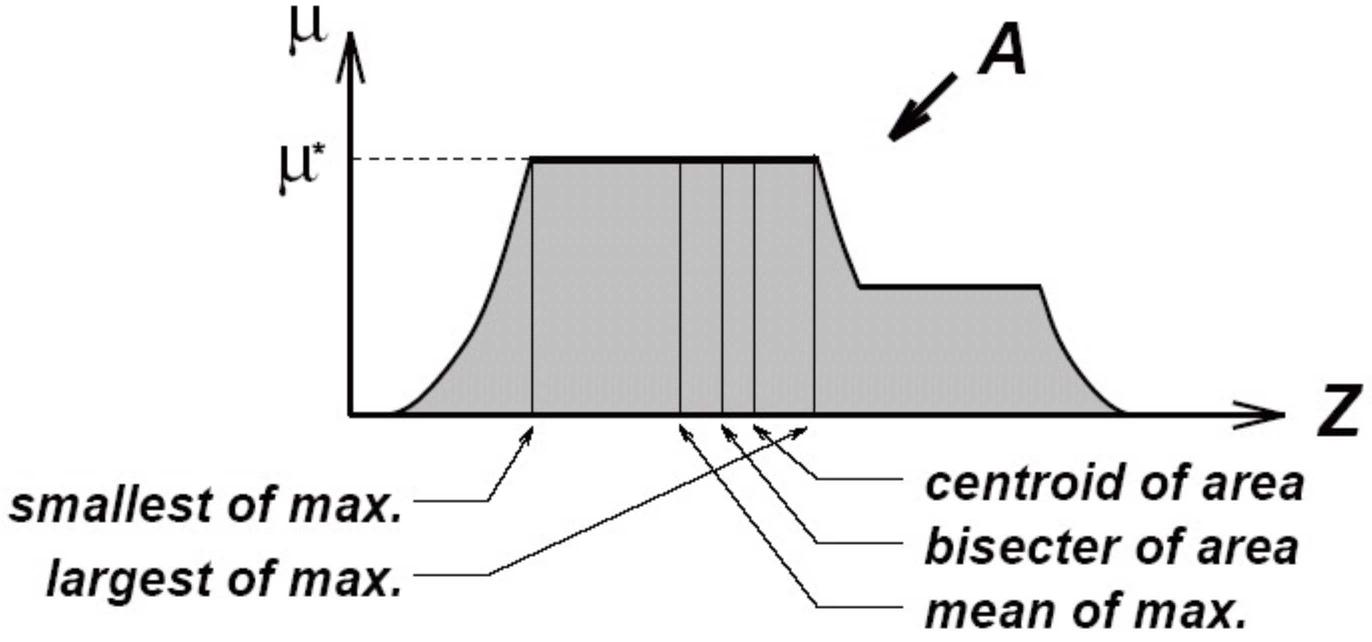


Defuzzifier

Since consequent is fuzzy, it has to be defuzzified

- Converts the fuzzy output of the inference engine to crisp using membership functions analogous to the ones used by the fuzzifier.
- Five commonly used defuzzifying methods:
  - Centroid of area (COA)
  - Bisector of area (BOA)
  - Mean of maximum (MOM)
  - Smallest of maximum (SOM)
  - Largest of maximum (LOM)

Defuzzifier



Takagi-Sugeno (TSK) Fuzzy Models

Fuzzy Rules of TSK Model

While antecedent is fuzzy, consequent is crisp

If x is A and y is B then $z = f(x, y)$

Fuzzy Sets

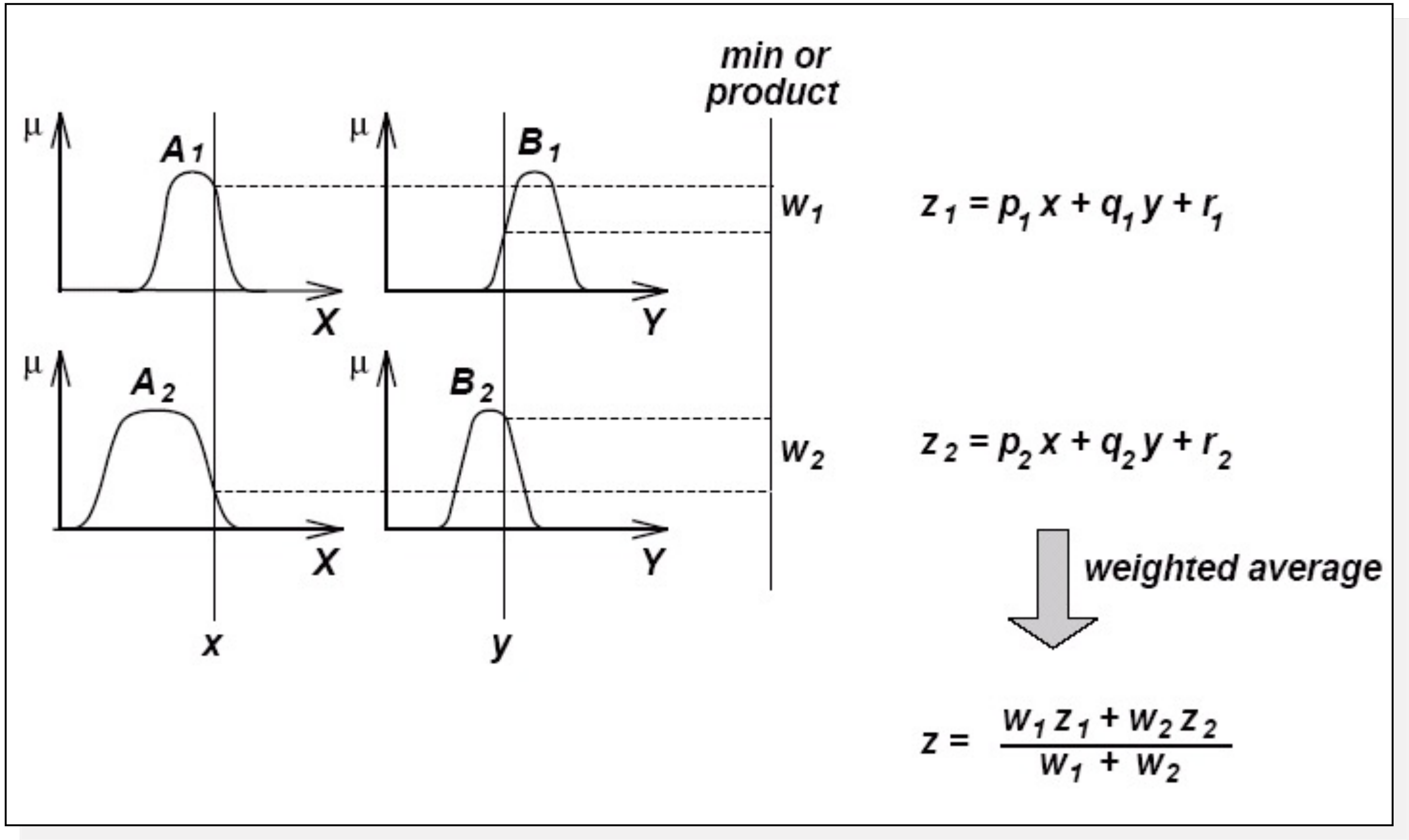
Crisp Function

The order of a Takagi-Sugeno type fuzzy inference system = the order of the polynomial used.

$f(x, y)$ is very often a polynomial function w.r.t. x and y.

The over all aggregated output will be obtained via Weighted Average Defuzzification Method

The Reasoning Scheme



Fuzzy Rules of TSK Model

- i-th rule can be represented

  Ri: x_1 *is* A^i *and* x_2 *is* B^i … *and* x_n *is* K^i *then*

$$z^i = a_0^i + a_1^i x_1 + \cdots + a_n^i x_n$$

- Weight of the i-th rule is

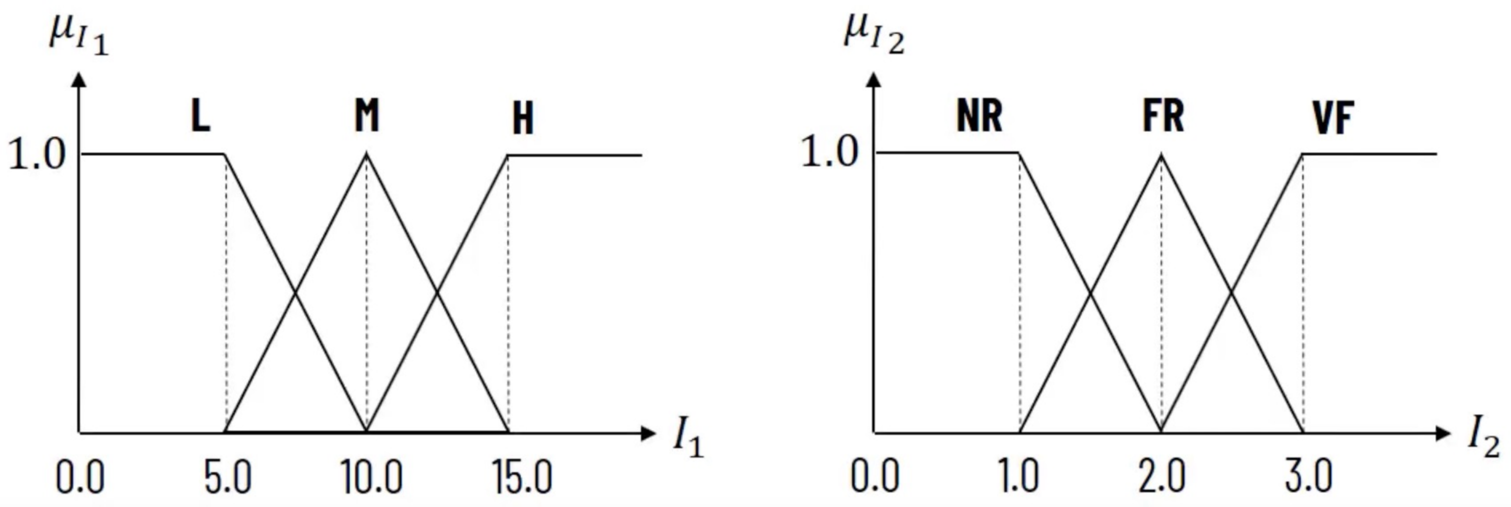
$$w_i = \mu_A^i(x_1) \times \mu_B^i(x_2) \times \cdots \times \mu_K^i(x_n)$$

- Combined control action

$$x^\star = \frac{\sum_{i=1}^{k} w^i z^i}{\sum_{i=1}^{k} w^i}$$

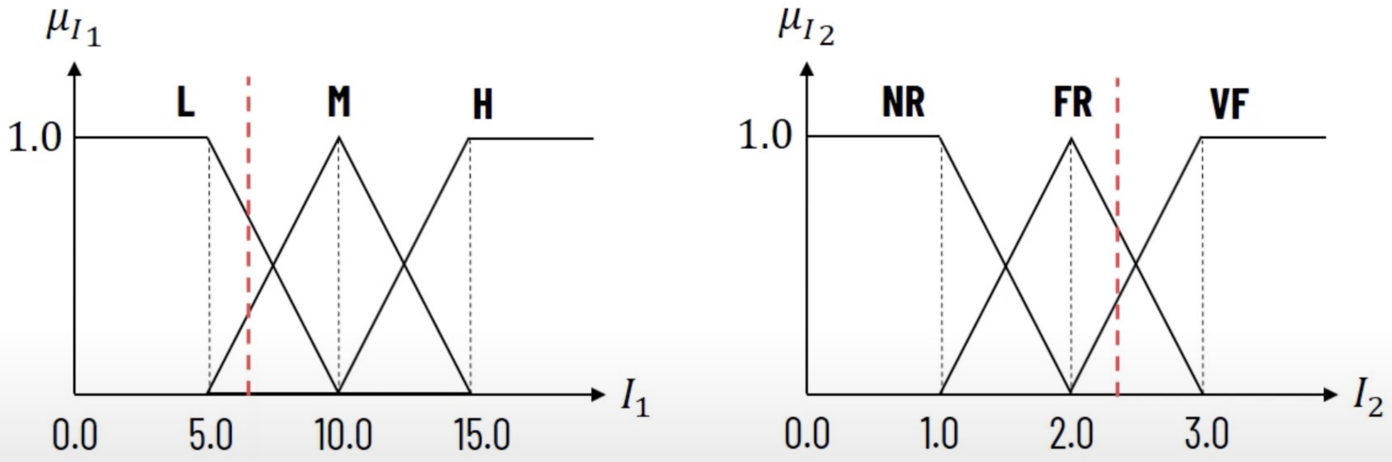
Example

- Solve it for Inputs $I_1 = 6.0$ and $I_2 = 2.2$



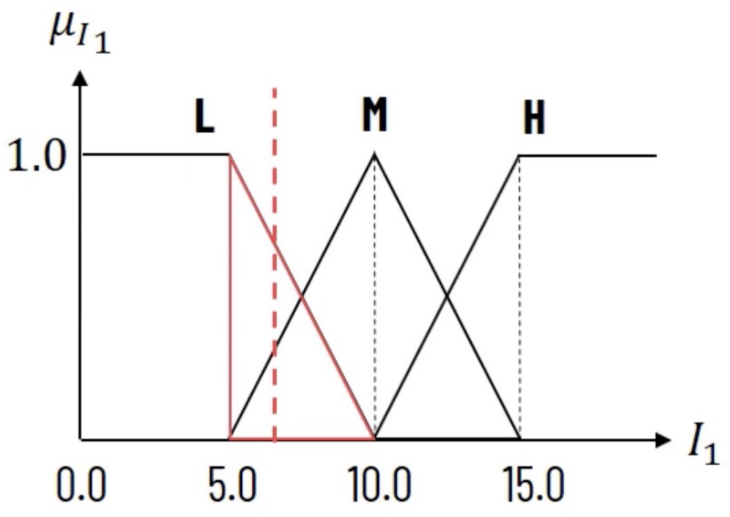
Example

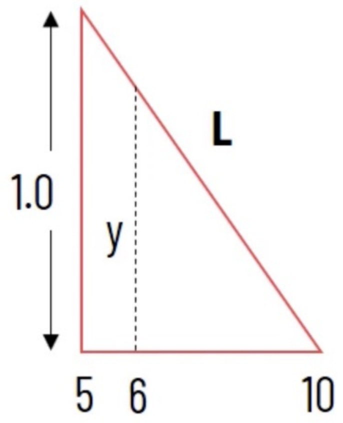
- Inputs: I_1 of 6.0 unit may be called **L** or **M**
- Inputs: I_2 of 2.2 unit may be called **FR** or **VF**



Example

- Using principle of similar triangle:





$$\frac{y}{1.0} = \frac{10 - 6}{10 - 5}$$

$$\Rightarrow y = \mu_L = 0.8$$

Example

- Similarly, $\mu_L = 0.8$, $\mu_M = 0.2$, $\mu_{FR} = 0.8$, $\mu_{VF} = 0.2$
- I_1 I in sets L and M, I_2 is in set FR and VF
- **Fired Set of Rules:**
  - I_1 is L and I_2 is FR
  - I_1 is L and I_2 is VF
  - I_1 is M and I_2 is FR
  - I_1 is M and I_2 is VF
- **Weights:**
  - $w^1 = \mu_L \times \mu_{FR} = 0.8 \times 0.8 = 0.64$
  - $w^2 = \mu_L \times \mu_{VF} = 0.8 \times 0.2 = 0.16$
  - $w^3 = \mu_M \times \mu_{FR} = 0.2 \times 0.8 = 0.16$
  - $w^4 = \mu_M \times \mu_{VF} = 0.2 \times 0.2 = 0.04$

Example

- Inputs: $I_1 = 6.0$ and $I_2 = 2.2$
- $y^1 = I_1 + 2I_2 = 6.0 + 2 \times 2.2 = 10.4$
- $y^2 = I_1 + 3I_2 = 6.0 + 3 \times 2.2 = 12.6$
- $y^3 = 2I_1 + 2I_2 = 2 \times 6.0 + 2 \times 2.2 = 16.4$
- $y^4 = 2I_1 + 3I_2 = 2 \times 6.0 + 3 \times 2.2 = 18.6$

i	w^i	y^i
1	0.64	10.4
2	0.16	12.6
3	0.16	16.4
4	0.04	18.4

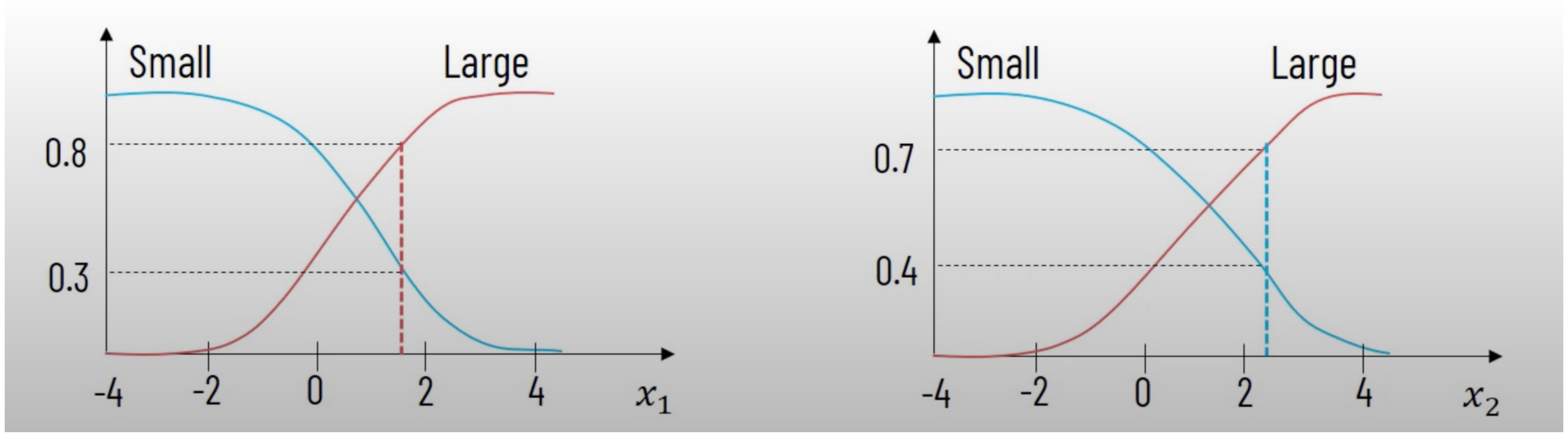
$$x^* = \frac{w^1y^1 + w^2y^2 + w^3y^3 + w^4y^4}{w^1 + w^2 + w^3 + w^4}$$

$$x^* = \frac{(0.64 \times 10.4) + (0.16 \times 12.6) + (0.16 \times 16.4) + (0.04 \times 18.6)}{0.64 + 0.16 + 0.16 + 0.04}$$

$$x^* = 12.04$$

Example

- Consider a two input – single output Sugeno model with 4 rules as:
  - **Rule 1:** IF x_1 is $small$ and x_2 is $small$ THEN $y_1 = -x_1 + x_2 + 1$
  - **Rule 2:** IF x_1 is $small$ and x_2 is $large$ THEN $y_2 = -x_2 + 3$
  - **Rule 3:** IF x_1 is $large$ and x_2 is $small$ THEN $y_3 = -x_1 + 3$
  - **Rule 4:** IF x_1 is $large$ and x_2 is $large$ THEN $y_4 = x_1 + x_2 + 2$
- Find the output when $x_1 = 1.5$ and $x_2 = 2.5$

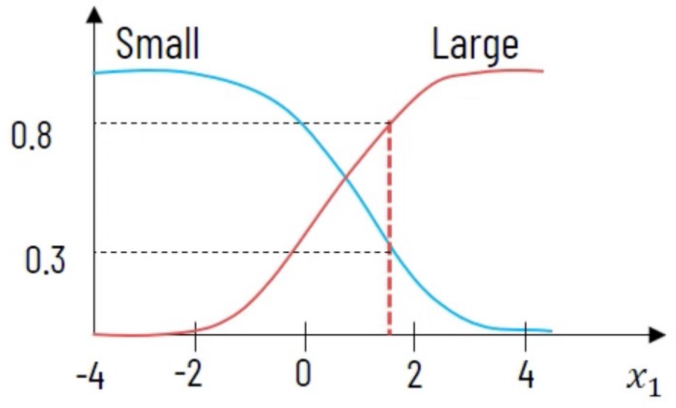


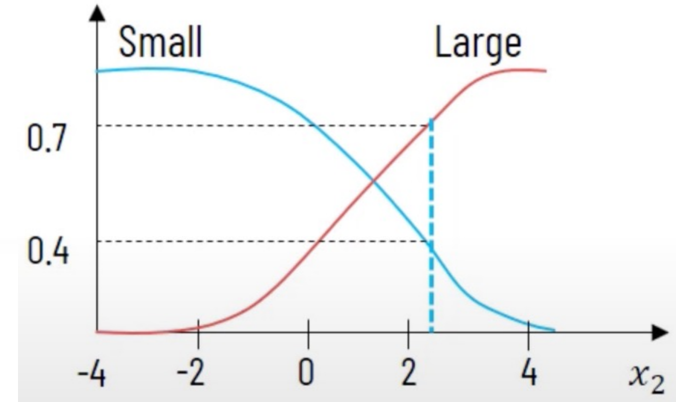
Example

- Here, $x_1 = 1.5, x_2 = 2.5$
- **Functional Consequents:**
  - $y^1 = -x_1 + x_2 + 1 = -1.5 + 2.5 + 1 = 2$
  - $y^2 = -x_2 + 3 = -2 + 3 = 0.5$
  - $y^3 = -x_1 + 3 = -1.5 + 3 = 1.5$
  - $y^4 = x_1 + x_2 + 2 = 1.5 + 2.5 + 2 = 6$

Example

- Here, $x_1 = 1.5$ and $x_2 = 2.5$
  - **Rule 1:** IF x_1 is $small$ and x_2 is $small$ THEN $y_1 = -x_1 + x_2 + 1$
  - **Rule 2:** IF x_1 is $small$ and x_2 is $large$ THEN $y_2 = -x_2 + 3$
  - **Rule 3:** IF x_1 is $large$ and x_2 is $small$ THEN $y_3 = -x_1 + 3$
  - **Rule 4:** IF x_1 is $large$ and x_2 is $large$ THEN $y_4 = x_1 + x_2 + 2$





- **Weights:**
  - $w^1 = min(\mu_{x_1}, \mu_{x_2}) = min(0.3, 0.4) = 0.3$
  - $w^2 = min(0.3, 0.7) = 0.3$
  - $w^3 = min(0.8, 0.4) = 0.4$
  - $w^4 = min(0.8, 0.7) = 0.7$

i	y^i	w^i
1	2	0.3
2	0.5	0.3
3	1.5	0.4
4	6	0.7

$$x^* = \frac{w^1 y^1 + w^2 y^2 + w^3 y^3 + w^4 y^4}{w^1 + w^2 + w^3 + w^4}$$

Fuzzy Rules of TSK Model

R: **if (*x* is** $\mu_A(x)$**) then z =** $p_0 + p_1$ x

$$z_1 = p_0 + p_1 x_1$$
$$z_2 = p_0 + p_1 x_2$$

$$p_1 = \frac{z_1 - z_2}{x_1 - x_2}, p_0 = z_1 - \frac{z_1 - z_2}{x_1 - x_2}$$

Mamdani or Sugeno

- **Mamdani method:**
  - It is widely accepted for capturing expert knowledge.
  - It allows us to describe the expertise in more intuitive, more human-like manner.
  - However, Mamdani-type fuzzy inference entails a substantial computational burden.
- **Sugeno method:**
  - It is computationally effective
  - It works well with optimization and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.
  - It avoids the time consuming methods of defuzzification that are needed for Mamdani model